An Attempt to Construct the Standard Model with Monopoles

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Abstract

We construct a model in which stable magnetic monopoles have magnetic charges that are identical to the electric charges on leptons and quarks and the colored monopoles are confined by strings in color singlets.

The similarities between magnetic monopoles connected by strings and quarks connected by chromoelectric flux tubes has been the basis for speculation over the last few decades that there may be a direct correspondence between the two [1]. This speculation is further fueled by the recent developments in supersymmetric theories where duality transformations can be found that relate the spectrum of particles in one theory to the spectrum of monopoles (and vice versa) in another dual theory [2]. In spite of the similarities in the pictures of quarks and monopoles, a specific model in which monopoles are confined in composites just as quarks are confined in baryons and mesons is lacking in the literature. It is the purpose of this paper to construct such a model. As a bonus, the model is found to contain other monopoles which do not get confined and which have a charge spectrum similar to that of the standard model leptons. The successful replication of the charge spectrum of the fermionic sector of the standard model by monopoles seems quite miraculous and leads us to speculate that perhaps the real world fermions are indeed monopoles of some grand unified bosonic theory. The difficulties likely to be encountered in developing such a scenario are discussed towards the end of the paper.

We start by listing the desirable features of the model that we are looking for. These are:

(i) The model should contain magnetic monopoles that are confined in twos and threes. This feature might be thought of as "color" confinement and so we might want the monopoles to carry $SU(3)$ charge.

(ii) The monopoles should also carry an $SU(2)$ charge which may be thought of as "weak" charge.

(iii) A monopole "cluster" (confined monopoles) should have the ability to carry net magnetic charge which may be identified with $U(1)$ "electromagnetic" charge.

A model that seems to satisfy all these requirements is one in which the (continuous) symmetry breaking pattern is:

$$\tilde{SU}(5) \to \tilde{U}(1) \tag{1}$$

where the final $\tilde{U}(1)$ is to be thought of as the dual electromagnetic symmetry group. One way to analyze the monopole and string content [3] of this symmetry breaking would be to construct all the embedded string solutions [4] and the "incarnations" of topological monopoles [5]. Another way, which

is simpler, is to imagine that the symmetry breaking in (1) occurs in stages. For example,

$$\tilde{SU}(5) \to [\tilde{SU}(3) \times \tilde{SU}(2) \times \tilde{U}(1)']/Z_6 \to [\tilde{SU}(3) \times \tilde{U}(1)]/Z_3 \to \tilde{U}(1) \quad (2)$$

The first step can be achieved if an $\tilde{SU}(5)$ adjoint Higgs (Φ_{24}) gets a vacuum expectation value (vev). The second symmetry breaking occurs by the vev of an $\tilde{SU}(2)$ fundamental (which can arise from an $\tilde{SU}(5)$ fundamental). The third symmetry breaking occurs if three $\tilde{SU}(3)$ adjoints but $\tilde{U}(1)$ singlets get vevs since it is known that the (generic) vevs of N adjoints of $SU(N)$ break the symmetry down to Z_N.

The reader would have surely noticed that the first two stages in (2) are nothing but the symmetry breaking pattern of minimal $SU(5)$ Grand Unification. Indeed this is true, and luckily for us, the monopoles in the model have been studied in great detail [6] together with their stability properties [7, 8].

The potential needed for the first symmetry breaking is:

$$V(\Phi_{24}) = -\frac{\mu^2}{2}\mathrm{Tr}[\Phi_{24}^2] + \frac{a}{4}(\mathrm{Tr}[\Phi_{24}^2])^2 + \frac{b}{2}\mathrm{Tr}[\Phi_{24}^4]$$

with the constraints: $a > 0 > -7b/15$. This potential is minimized by the vev:

$$< \Phi_{24} >= v \ \mathrm{diag}\left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}, -\frac{1}{2}, -\frac{1}{2}\right) \ .$$

which is annihilated by the generators of $\tilde{SU}(3)$ acting on the upper left 3×3 elements, $\tilde{SU}(2)$ acting on the lower right 2×2 elements and by the $\tilde{U}(1)$ generator given by

$$Q_1 = \frac{1}{v} < \Phi_{24} >= \mathrm{diag}\left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}, -\frac{1}{2}, -\frac{1}{2}\right) \ .$$

But there are 3 group elements that are shared between $\tilde{SU}(3)$ and $\tilde{U}(1)'$ which correspond to the center of $\tilde{SU}(3)$ and are:

$$\exp\left[i2\pi n \mathrm{diag}\left(\frac{1}{3}, \frac{1}{3}, -\frac{2}{3}, 0, 0\right)\right] = \exp\left[i2\pi n Q_1\right], \quad n = 0, 2, 4 \ .$$

There are also 2 group elements that are shared between $\tilde{SU}(2)$ and $\tilde{U}(1)'$ which correspond to the center of $\tilde{SU}(2)$ and these are:

$$\exp\left[i2\pi n \mathrm{diag}\left(0, 0, 0, \frac{1}{2}, -\frac{1}{2}\right)\right] = \exp\left[i2\pi n Q_1\right], \quad n = 0, 3 \ .$$

So to avoid overcounting the discrete group $Z_3 \times Z_2$ which is the center of $\tilde{SU}(3) \times \tilde{SU}(2)$, we must mod out the unbroken continuous symmetries by Z_6.

The monopoles formed at the first symmetry breaking are given by the first homotopy of the unbroken symmetry group. This means that we have to construct closed paths on the group manifold that are incontractible. Each class of homotopically inequivalent paths, leads to a distinct monopole. Clearly, an incontractible path is one which wraps around the $\tilde{U}(1)'$ and this can be written as:

$$\exp[i6Q_1 s], \quad s \in [0, 2\pi] \tag{3}$$

where Q_1 is the generator of $\tilde{U}(1)'$ and s is a parameter that labels points on the closed path. But there are other incontractible paths present too - for example, there is a path that goes through $\tilde{U}(1)'$, $\tilde{SU}(2)$ and $\tilde{SU}(3)$. This path may be written in the form of (3) but with $6Q_1$ replaced by

$$Q_m = Q_3 + Q_2 + Q_1$$

where, the $\tilde{SU}(3)$ and $\tilde{SU}(2)$ charge operators are

$$Q_3 = \text{diag}\left(-\frac{1}{3}, -\frac{1}{3}, \frac{2}{3}, 0, 0\right)$$

$$Q_2 = \text{diag}\left(0, 0, 0, \frac{1}{2}, -\frac{1}{2}\right) \ .$$

The monopole corresponding to this path is the "minimal" monopole and all other monopoles in the model can be thought of as multiply charged monopoles of this variety. So the monopoles in the model have charge $n \times Q_m$, where n is any integer. But the $SU(3)$ charge is a Z_3 charge and the $SU(2)$ charge is a Z_2 charge. Hence, the winding n monopole has magnetic charge:

$$Q_m^{(n)} = n_3 Q_3 + n_2 Q_2 + n_1 Q_1 \ ,$$

where, $n_3 = n(\text{mod } 3)$, $n_2 = n(\text{mod } 2)$ and $n_1 = n$. The first four columns of Table 1 display the quantum numbers of monopoles with different winding numbers.

Table 1: Quantum numbers on monopoles with winding $n \leq 6$ and charges on standard model fermions in units of the charges on $(u,d)_L$.

n	n_3	n_2	n_1		$SU(3)_c$	$SU(2)_L$	$U(1)_Y$
1	1	1	1	$(u,d)_L$	1	1	+1
2	2	0	2	d_R	1	0	-2
3	0	1	3	$(\nu,e)_L$	0	1	-3
4	1	0	4	u_R	1	0	+4
5	2	1	5	-	-	-	-
6	0	0	6	e_R	0	0	-6

Even though monopoles of arbitrary charge are allowed topologically, they may not exist for dynamical reasons or may be unstable to fragmenting into monopoles of smaller winding. Gardner and Harvey [7] have argued that monopoles in the first stage of symmetry breaking are stable only when $n = \pm1, \pm2, \pm3, \pm4$, and, ±6 provided $\mu_0 << \mu_8 = \mu_3/2$ where μ_0, μ_3 and μ_8 are the masses of the singlet, triplet and octet components of Φ_{24}. A crucial observation here is that monopoles with $n = \pm5$ are unstable.

In the last four columns of Table 1 we show the $SU(3)_c$, $SU(2)_L$ and $U(1)_Y$ charges on the leptons and quarks of the standard model in units of the charges on $(u,d)_L$. A comparison of the left- and right-hand sections of Table 1 suggests the following identifications:

$$\begin{aligned} (u,d)_L &\rightarrow n = +1 \\ d_R &\rightarrow n = -2 \\ (\nu,e)_L &\rightarrow n = -3 \\ u_R &\rightarrow n = +4 \\ e_R &\rightarrow n = -6 \ . \end{aligned} \tag{4}$$

The monopoles with non-trivial $\tilde{SU}(3)$ and $\tilde{SU}(2)$ charges are three- and two-fold degenerate respectively. It is simplest to see this for the monopoles with non-trivial $\tilde{SU}(2)$ charge but vanishing $\tilde{SU}(3)$ charge [9]. These monopoles exist due to incontractible closed paths that are entirely in $[\tilde{SU}(2) \times \tilde{U}(1)']/Z_2$.

Then consider the two incontractible paths:

$$g_\pm(s) = \exp\left[is\left(\frac{\mathbf{1}\pm\tau_3}{2}\right)\right]$$

where, $s \in [0, 2\pi]$ is the parameter labeling the path. It can be checked that the path g_+ can be deformed into g_- and so the paths are topologically equivalent. This would lead one to think that there is only one monopole. But now consider what happens when the $[\tilde{SU}(2) \times \tilde{U}(1)']/Z_2$ gets broken as in the last stage of (2). Suppose the generator of the final unbroken $\tilde{U}(1)$ is $Q = (\mathbf{1}+\tau_3)/2$ as is conventionally taken in the standard electroweak symmetry breaking. Then the g_+ monopole will continue to have a long range $\tilde{U}(1)$ magnetic field but the g_- monopole magnetic field will get screened. So we should think of the monopoles with non-trivial $\tilde{SU}(2)$ charge as being two-fold degenerate with the degeneracy being lifted in the last stage of (2). Similarly, we should think of the monopoles with non-trivial $\tilde{SU}(3)$ charge as being three-fold degenerate.

The two-fold degeneracy of monopoles with non-vanishing n_2 is brought out in (4) since we have to identify fermion doublets with the $n = +1$ and $n = -3$ monopoles. Similarly, the three-fold degeneracy of monopoles with non-vanishing n_3 means that these monopoles should come in the fundamental representation of an $SU(3)$. The reason that we sometimes have to choose to identify the fermions with antimonopoles ($n < 0$) rather than monopoles is so that the hypercharge assignments tally. (The $\tilde{SU}(2)$ charge on the monopole is a Z_2 charge and so the sign is not important. The $\tilde{SU}(3)$ charge is a Z_3 charge and so -2 is the same as +1.) Remarkably, this identification yields the correct $\tilde{SU}(3)$ charges of the standard model fermions. It also seems somewhat of a miracle that the $n = \pm 5$ monopoles are unstable at the same time that we do not observe any fermions of hypercharge equal to 5/6.

Is the correspondence between the standard model fermions and stable monopoles some group theory magic? This cannot be entirely true since the stability of a multiply charged monopole also depends on the dynamical requirement that $\mu_0 << \mu_8$. However, it is true that the instability of the $n = \pm 5$ monopole is independent of the choice of parameters since it can always fragment into two monopoles of winding numbers ± 2 and ± 3 [7].

The second symmetry breaking in (2) corresponds to the electroweak symmetry breaking and this is known not to have any topological strings. Hence, as discussed in Ref. [9, 7], the monopoles carrying $\tilde{SU}(2)$ charge will

not get confined by strings during this symmetry breaking [10]. The $\tilde{SU}(2)$ gauge fields will get screened and the confined monopole clusters can only carry long range $\tilde{SU}(3)$ and $\tilde{U}(1)$ magnetic fields. The electroweak $Z-$string and Nambu's monopoles [11] will also be present in this symmetry breaking. When dualized, the electroweak monopole will appear as an electrically charged confined particle.

In the last symmetry breaking stage, the $\tilde{SU}(3)$ factor breaks down to Z_3 which is the center of the group. This symmetry breaking has non-trivial first homotopy:

$$\pi_1(\tilde{SU}(3)/Z_3) = Z_3$$

and so the symmetry breaking yields topological Z_3 strings. The Z_3 strings produced at this stage are deformable to the vacuum if we allow excitations of the $\tilde{SU}(5)$ degrees of freedom. This means that the Z_3 strings can end on monopoles which carry $\tilde{SU}(3)$ charge. These monopoles are precisely those that correspond to the quarks ($n = 1, -2$ and 4) and the quark monopoles are confined by Z_3 strings in chromomagnetic singlets. But the cluster can still have $\tilde{U}(1)$ charge.

Assuming that this model can be suitably dualized, we would like to identify the confined monopole clusters with the hadrons and the unconfined monopoles with the leptons. For example, in this picture, the proton would be identified with three $n = 1$ monopoles that have been confined with net $\tilde{U}(1)$ charge equal to $+1$. And the unconfined $n = +3$ monopole would be identified with the left-handed anti-neutrino and positron doublet. Proton decay might correspond to the collapse of 3 confined $n = 1$ monopoles to form a single $n = +3$ monopole.

The correspondence between the $\tilde{SU}(5)$ monopoles and the standard model fermions suggests that, perhaps "Grand Unification" should be based on an $\tilde{SU}(5)$ symmetry group with only a bosonic sector and the presently observed fermions are really the monopoles of that theory. However, as we now discuss, there are numerous challenges that need to be addressed before this conjecture can be tested.

The first question is that why should the monopoles be fermions and not bosons? This problem may already have been resolved due to the discovery that isopin can lead to spin [12]. The idea is that a bound state of a charged boson and a monopole forms a dyon that can have integer or half-integer spin if the *iso*spin of the free boson is integer or half-integer respectively.

Goldhaber has shown that dyons with half-integer spin also obey Fermi-Dirac statistics [13]. These results may be relevant to our construction but there is also a problem in this approach. The four (degenerate) dyons that result from the bound state of a monopole and a charged boson can have magnetic and electric charges $(\pm 1, \pm 1/2)$ in suitable units. A duality rotation cannot result in all these dyons having pure electric charges. In this picture we would get the standard model fermions as well as light magnetic monopoles.

Another scheme to convert monopoles to dyons is by the introduction of a θ term in the $\tilde{SU}(5)$ action. This term would be proportional to $\tilde{\theta} F^a_{\mu\nu} \tilde{F}^{\mu\nu a}$ where $F^a_{\mu\nu}$ and $\tilde{F}^{\mu\nu a}$ are the $\tilde{SU}(5)$ field strengths and their duals. Note that we have chosen to denote the coefficient of the term by $\tilde{\theta}$ and not by θ because we are assuming that the model will ultimately be dualized and that the θ in the standard model will be different from $\tilde{\theta}$. Witten [14] showed that the presence of such a term in the action will convert monopoles into dyons with electric charge $\tilde{e}\tilde{\theta}/2\pi$ where $\tilde{e}$ is the smallest unit of electric charge in the original (undualized) model. As the spin of a dyon is related to the angular momentum in the long range fields, it seems reasonable to assume that the dyon in the case when $\tilde{\theta} = \pi$ should also be treated as a spin 1/2 object obeying Fermi-Dirac statistics. The advantage in this approach seems to be that, since the θ term is CP violating, there are only two degenerate dyons present with magnetic and electric charges $\pm(1, 1/2)$. And now a duality rotation can convert these dyons into particles carrying only electric charges.

Note that the scheme presented in this paper is based on a different philosophy than the scheme used in supersymmetric duality. Were we to start out with a supersymmetric theory, our model would already contain both bosons and fermions in supersymmetric multiplets. The monopole solutions would be in addition to all the supersymmetric particles. In the present scheme, we have started out with only bosons - hence, the model is necessarily non-supersymmetric - and the monopoles are identified with the fermionic sector of another theory - in this case, the standard model.

An aspect we have not addressed is the three family structure of the standard model fermions. It is probably possible to get three families of monopoles by increasing the number of scalar fields in the model and perhaps introducing new symmetries under the exchange of these fields [15]. For example, one could consider the case when the first symmetry breaking occurs by the parallel (but not necessarily equal) vevs of 3 different adjoints

of $\tilde{SU}(5)$. Indeed, this might be desirable since we also need three $\tilde{SU}(3)$ adjoints to get vevs during the last stage of symmetry breaking in (2) and each $\tilde{SU}(3)$ adjoint could come from an $\tilde{SU}(5)$ adjoint. If this does lead to three families of monopoles - and this is something that needs to be investigated - it would relate the number of families to the number of colors in QCD.

A vexing problem is the connection of $\tilde{SU}(2)$ and chirality and this remains one of the many open questions. Another problem is to study the stability of the monopoles at strong couplings and to actually dualize the model. Only when this is done, will it be possible to determine the masses and interactions of the particles (dualized monopoles).

Another aspect that we have not addressed is if the picture can have any cosmological relevance? Can it be that only bosonic particles filled the universe at some epoch and, after some phase transitions, got replaced by monopoles which today look like the fermionic sector of the standard model?

Within the framework of this model, it seems that the bosonic sector of the standard model will be replaced by a "dyonic" sector since, after the duality transformation, the original electrically charged particles (Higgses and gauge fields of $\tilde{SU}(5)$) will get a magnetic description. A prediction of this picture may be one that we have alluded to earlier - when the model is dualized, Nambu's electroweak monopoles [11] would appear as electric charges that are confined by $Z-$electric flux tubes. These would appear as new confined particles. Also, we expect that the standard model fermions should resemble solitons (monopoles or dyons) at short distance scales. Then, for example, head-on collisions of particles at very high energies should lead to 90^o scattering [16].

Finally, we should point out that a spectrum of monopoles similar to the standard model would also result from other symmetry breaking schemes. This is because the spectrum of stable monopoles only depends on the first homotopy of $[\tilde{SU}(3) \times \tilde{SU}(2) \times \tilde{U}(1)']/Z_6$ and any symmetry breaking pattern starting from a simply connected group and with this group as an intermediate symmetry group will at least contain the monopoles corresponding to the standard model. Different schemes could, however, contain extra monopoles that would need to be included in the spectrum. The $\tilde{SU}(5)$ model we have considered here is the minimal model that contains all the monopoles that correspond to the standard model fermions.

To conclude, we have found a correspondence between stable monopoles in an $\tilde{SU}(5)$ theory and the standard model fermions. The colored monopoles

in this model are confined just as quarks are confined in color singlets. To take the correspondence further and claim an equivalence between monopoles and standard model fermions is a much more difficult task. But, if successfully done, it could shed light on several aspects of the standard model such as - the charge spectrum of fermions, why the fermions appear in fundamental representations, the replication of the fermions in three families, the confinement of quarks and other issues. To help us out in this daunting task are the many beautiful ideas that have been proposed over the last two decades.

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